

# Boolean Algebra Exercises

pp 1-2 Reference

pp 3-6 Exercises

## Boolean Algebra - Operations and Constants

- $A \text{ AND } B = A \wedge B = AB$
- $A \text{ OR } B = A \vee B = A+B$
- $\text{NOT } A = \neg A = A'$
- $\text{TRUE} = T = 1$
- $\text{FALSE} = F = 0$

## Boolean Algebra - Identities

- $A \cdot \text{True} = A$
- $A \cdot \text{False} = \text{False}$
- $A \cdot A = A$
- $(A')' = A$
- $A + A' = \text{True}$
- $A \cdot A' = \text{False}$
- $A + \text{True} = \text{True}$
- $A + \text{False} = A$
- $A + A = A$

## **Commutative, Associative, and Distributive Laws**

- $AB = BA$  (Commutative)
- $A + B = B + A$
- $A(BC) = (AB)C$  (Associative)
- $A + (B + C) = (A + B) + C$
- $A(B + C) = (AB) + (AC)$  (Distributive)
- $A + (BC) = (A + B)(A + C)$

## **DeMorgan's Laws**

- $(A + B)' = A'B'$
- $(AB)' = A' + B'$

## Example: Proving Identities

- Using truth tables, prove:

- $A + A' = \text{True}$

- $A \cdot A' = \text{False}$

| A | A' | A + A' |
|---|----|--------|
| F | T  | T      |
| T | F  | T      |

| A | A' | A • A' |
|---|----|--------|
| F | T  | F      |
| T | F  | F      |

## (One of the) Associative Laws

- Using truth tables, prove

$$A(B C) = (A B) C$$

| A | B | C | BC | A(BC) | AB | (AB)C |
|---|---|---|----|-------|----|-------|
| F | F | F | F  | F     | F  | F     |
| F | F | T | F  | F     | F  | F     |
| F | T | F | F  | F     | F  | F     |
| F | T | T | T  | F     | F  | F     |
| T | F | F | F  | F     | F  | F     |
| T | F | T | F  | F     | F  | F     |
| T | T | F | F  | T     | T  | T     |
| T | T | T | T  | T     | T  | T     |

## (One of the) Distributive Laws

- Using truth tables, prove

$$A(B + C) = (AB) + (AC)$$

| A | B | C | B + C | A(B + C) | AB | AC | (AB) + (AC) |
|---|---|---|-------|----------|----|----|-------------|
| F | F | F | F     | F        | F  | F  | F           |
| F | F | T | T     | F        | F  | F  | F           |
| F | T | F | T     | F        | F  | F  | F           |
| F | T | T | T     | F        | F  | F  | F           |
| T | F | F | F     | F        | F  | F  | F           |
| T | F | T | T     | F        | F  | F  | F           |
| T | T | F | T     | T        | T  | F  | T           |
| T | T | T | T     | T        | T  | T  | T           |

## Proving DeMorgan's Laws (a)

- Using truth tables, prove  $(A + B)' = A'B'$

| A | B | A + B | (A + B)' |
|---|---|-------|----------|
| F | F | F     | T        |
| F | T | T     | F        |
| T | F | T     | F        |
| T | T | T     | F        |

| A | B | A' | B' | A'B' |
|---|---|----|----|------|
| F | F | T  | T  | T    |
| F | T | T  | F  | F    |
| T | F | F  | T  | F    |
| T | T | F  | F  | F    |

## Proving DeMorgan's Laws (b)

- Prove the 2<sup>nd</sup> of DeMorgan's Laws:

$$(AB)' = A' + B'$$

| A | B | AB | (AB)' |
|---|---|----|-------|
| F | F | F  | T     |
| F | T | F  | T     |
| T | F | F  | T     |
| T | T | T  | F     |

| A | B | A' | B' | A'+B' |
|---|---|----|----|-------|
| F | F | T  | T  | T     |
| F | T | T  | F  | T     |
| T | F | F  | T  | T     |
| T | T | F  | F  | F     |

## Exercise: $A(A + B)$

| A | B | A + B | A(A + B) |
|---|---|-------|----------|
|---|---|-------|----------|

What have we proved in this table?

## Exercise: Boolean Algebra

- Exercise - Using the Distributive Property, Identities, and your result from the previous exercise, prove:

$$\begin{aligned} & \dots A + (AB) = A \end{aligned}$$

$$\begin{aligned} & \dots A + (AB) \end{aligned}$$

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## Exercise: Using DeMorgan's Laws

- Exercise – Using Boolean Algebra, including DeMorgan's Laws, prove:

$$\begin{aligned} & \dots (A'B)' = A + B' \end{aligned}$$

$$\begin{aligned} & \dots (A'B)' \end{aligned}$$

$$= ((A')B)' \quad \text{(add parentheses)}$$

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